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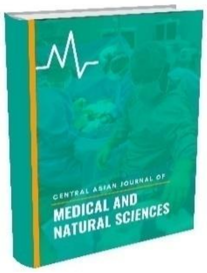

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Application of Mathematical Modeling to Determine the Growth in Weight of a Fish Species

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Abstract: The quantity and quality of food available, temperature, oxygen, and other water quality factors are all important factors in fish growth. Every animal grows in length and weight throughout its life, establishing a standard relationship (length-weight relationship). The length-weight relationship is a standard method used in fishery assessment studies because it provides useful information about fish growth. The length-weight relationship connects mathematics and the evaluation of variation between the two variables, length and weight. In this research, we have developed a mathematical model to predict the weight variation of fish species over a period, given a constant supply of adequate food under necessary conditions. The developed Bernoulli mathematical model was solved directly, and the function was coded using Wolfram Mathematica, version 12, where the energy supplied by the food and the surface area of the fish, and the energy used by the fish seeking food and its weight were investigated. In conclusion, this research is very useful for scientists and fisheries in managing and producing healthy fish for human consumption.

Keywords: Modeling, Fish, Growth, Length, Species, Bernoulli, Application.

1. INTRODUCTION

Optimal feeding schedules and harvesting policies are critical in fish farming because profitability is critical in fish farming, as it is in any other business. Fish farmers must have access to well-balanced nutritive and cost-effective feeds, as well as up-to-date information on farm feed management techniques, in order to realize high profit margins, as it is also a fact that providing a proper diet and feeding schedule for your fish will ensure better growth, disease resistance, vibrant colors, and long healthy lives. Farmers who invest in a specific fish species want to know how long it takes for it to reach the slaughter stage and reproduce so that they can plan their production and profit Bernaduce (2006).

In their study, Benaduce *et al.* (2006) emphasized the usefulness of the application of mathematical models in the field of biological sciences to help predict specific data. Their study focused on the relationship between weight and age of a fish species, as it has recently been seen as a topic of interest in fish biology. They also emphasized that the models can be utilized to simulate better handling of fish stock or to increase one's knowledge about biological interactions in the ecosystem. The mathematical models applied in this model also revealed that the silver catfish reaches its maximum weight around the age of 18 years, whereas the male of this species reaches its maximum weight around the age of 12 years. Furthermore, it has been demonstrated that male and female silver catfish grow similarly up to their sixth year of life. Males continue to grow after the sixth year, but at a slower rate than females. It is also clear that the maximum weight of male silver catfish is nearly half that of females. This clearly shows that mathematical models are vital tools in fish farm management.

Auger and Lett (2015) study revealed that marine resource management must be based on decision-making tools that allow managers and decision-makers to take measures for fisheries conservation and optimal exploitation. Thus, they added, mathematical modeling enables the development of such tools for predicting the effects of coastal development and fishing control measures.

In their study, Biswas *et al.* (2017) developed and tested a nonlinear mathematical model of fishery management to better understand the dynamics of a fishery resource system in an aquatic environment divided into two zones: free fishing and reserve fishing. They examined the model by determining the existence of equilibrium points: biological and bionomic; the dynamical behavior of equilibrium points; and the system's stability and instability conditions. The numerical simulations used to establish the presented analytical results demonstrated the behavior of this dynamical model of marine fishery management.

Dubey *et al.* (2003) proposed and examined a mathematical model to investigate the dynamics of a fishery resource system in an aquatic environment divided into two zones: a free fishing zone and a reserve zone where fishing was strictly prohibited. The system's biological and bionomic equilibria were determined, and criteria for local stability, instability, and global stability were also developed. It is demonstrated that even if a fishery is continuously exploited in the unreserved zone, fish populations in the habitat can be maintained at an appropriate equilibrium level. The Pantryagin's Maximum Principle was used to discuss an optimal harvesting policy.

In optimal control theory, Pontryagin's maximum principle is used to find the best possible control for moving a dynamical system from one state to another, especially when the state or input controls are constrained. It states that any optimal control must solve the so-called Hamiltonian system, which is a two-point boundary value problem plus a maximum condition of the control Hamiltonian, in addition to the optimal state trajectory.

Based on the literature reviewed above and to the best of our knowledge, we have proposed a mathematical model to predict the weight variation of a fish species over time, given a constant supply of adequate food and constant condition factors.

2. FORMULATION OF MATHEMATICAL MODEL

Before we formulate the mathematical model to study the growth of the fish species, let's see Fig. a

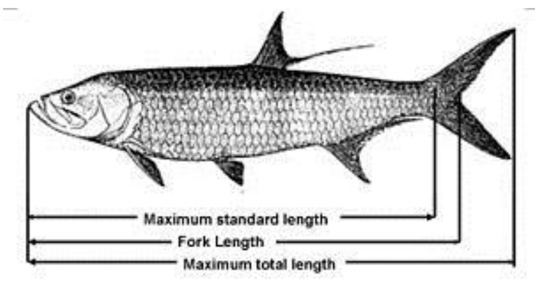

Fig. a Picture Showing the Fish Species

The weight of a fish is the weight at a given total length for the specific species of that fish. Experiment and observation indicate that the weight of a fish is proportional to the cube of its length, thus establishing a relationship between length (L) and weight (W) in fishes of the form.

$$W = kl^b \tag{1}$$

In equation (1) above, the length-weight parameters k and b are to be estimated using the available length-weight data on fish samples. However, each species of fish has a unique length-weight relationship or length-weight parameter values. It may also differ between sexes as well as between stocks or those from different geographical regions. The parameter k is a scaling coefficient for the fish species' weight at length. The parameter b is a shape parameter for the fish species' body form.

In theory, the exponent b in equation(1) is roughly equal to three because the volume of a three-dimensional (regularly shaped solid) object is roughly proportional to the cube of its length. Literally, length is one-dimensional, and weight depends on volume, which is three-dimensional; thus, the weight of a fish is assumed to be roughly equivalent to the cube of its length; thus, we write equation (1) as follows:

$$W = kl^3 \tag{2}$$

In addition, numerous studies on the subject have been published. Field studies have also revealed that a fish's surface area (A) is proportional to the square of its length, as shown by the following equation.

$$A = pl^2 \tag{3}$$

The coefficient p in the preceding equation, equation(3) is proportionality constant. Other related literature on this topic also revealed that the energy (E_0) expended by a fish in seeking food is proportional to its weight, while the energy (E_1) provided by the food is proportional to its surface area.

3. MODEL DEVELOPMENT

A mathematical model uses mathematical language, symbols, and concepts to describe the behavior of a real-world system. It is an abstract model that uses mathematical language to explain a system's behavior, Bunonyo and Awomi (2022) and Bunonyo *et al*. (2022). According to Eykhoff (1974), a mathematical model is a representation of the main elements of an existing system (or a system to be developed) that provides usable knowledge of that system.

To develop a model to determine the variation in weight of a fish over time, we assume that the rate of increase in weight of the fish species is proportional to the difference between the two energies (i.e., the energy used in seeking food (E_0) and the energy provided by the food (E_1)). With this assumption, we can now construct an equation (model) that depicts the variation of the weight of the fish species over time, as shown below.

$$\frac{dW}{dt} = \kappa(E_1 - E_0) \tag{4}$$

It is obvious from equation (4) that the variable W, defined by $W = kl^3$ is the weight of the fish species, where l is the length of the fish species and k is a proportionality constant that serves as a scaling coefficient for the weight at the length of the fish species and is usually determined by available length-weight data or by an experimental process. Also the energy used by the fish in seeking food which is proportional to the weight of the fish is given by

$$E_0 = NW \tag{5}$$

While the energy provided by the food which is proportional to the surface area of the fish is given by

$$E_1 = GA \tag{6}$$

where κ, N and G in equations(4), (5) and (6) are constants of proportionality and the variable A which is the surface area of the fish is defined by $A = pl^2$.

4. MODEL SOLUTION

To get the analytical solution of the model in equation (4) above, we insert equations (5) and (6) into equation(4) which yields the following equation

$$\frac{dW}{dt} = \kappa(GA - NW) \tag{7}$$

Since the weight (W) is proportional to the cube of the length of the fish, defined by

$$W \propto l^3 \tag{8}$$

$$W^{2/3} \propto l^2 \tag{9}$$

But the surface area of the fish species is proportional to the square of the length of the fish and is defined by the following

$$A \propto l^2 \tag{10}$$

Now, from equations(9) and (10) it can be seen that the surface area A of the fish species can also be written in the form seen below

$$A = W^{2/3} \tag{11}$$

Then equation (7) becomes the following

$$\frac{dW}{dt} = \kappa(MW^{2/3} - NW) \quad \forall \kappa, M, N \geq 1 \tag{12}$$

Setting $\beta_0 = \kappa M$ and $\beta_1 = \kappa N$ where M and N, are the constants of proportionality, then equation (12) becomes

$$\frac{dW}{dt} = \beta_0 W^{2/3} - \beta_1 W \tag{13}$$

To solve the Bernoulli's differential equation in equation (13) above, Let

$$v = W^{1-2/3} = W^{1/3} \tag{14}$$

Thus, we get the following equation

$$\frac{dv}{dt} = \frac{1}{3} W^{-2/3} \frac{dW}{dt} \tag{15}$$

Now from (13) and(15), we get the following equation

$$\frac{dv}{dt} = \frac{1}{3} W^{-2/3}\left(\beta_0 W^{2/3} - \beta_1 W\right) \tag{16}$$

$$\frac{dv}{dt} = \frac{\beta_0}{3} - \frac{\beta_1}{3} W^{1/3} \tag{17}$$

But $v = W^{1/3}$ from(14), hence we get the following equation

$$\frac{dv}{dt} = \frac{\beta_0}{3} - \frac{\beta_1}{3} v \tag{18}$$

$$\frac{dv}{dt}+\frac{\beta_1}{3}v=\frac{\beta_0}{3} \tag{19}$$

Since equation (13) has been reduced to a first order linear ordinary differential equation in equation(19), we can now find an integrating factor, say μ to solve the differential equation, thus we let

$$\mu=e^{\int\frac{\beta_1}{3}dt} \tag{20}$$

$$\mu=e^{\frac{\beta_1}{3}t} \tag{21}$$

Multiplying equation (19) by the value of μ yields the following:

$$\frac{dv}{dt}e^{\frac{\beta_1}{3}t}+\frac{\beta_1}{3}ve^{\frac{\beta_1}{3}t}=\frac{\beta_0}{3}e^{\frac{\beta_1}{3}t} \tag{22}$$

Writing the left hand side of equation (22) in a compacted form yields

$$\frac{d}{dt}\left(ve^{\frac{\beta_1}{3}t}\right)=\frac{\beta_0}{3}e^{\frac{\beta_1}{3}t} \tag{23}$$

Integrating both sides of equation (23) with respect to t yields

$$\int\frac{d}{dt}\left(ve^{\frac{\beta_1}{3}t}\right)dt=\int\frac{\beta_0}{3}e^{\frac{\beta_1}{3}t}dt \tag{24}$$

$$ve^{\frac{\beta_1}{3}t}=\frac{\beta_0}{3}\int e^{\frac{\beta_1}{3}t}dt \tag{25}$$

$$ve^{\frac{\beta_1}{3}t}=\frac{\beta_0}{3}\left(\frac{e^{\frac{\beta_1}{3}t}}{\beta_1/3}\right)+C \tag{26}$$

$$ve^{\frac{\beta_1}{3}t}=\frac{\beta_0}{3}.\frac{3}{\beta_1}\left(e^{\frac{\beta_1}{3}t}\right)+C \tag{27}$$

$$ve^{\frac{\beta_1}{3}t}=\frac{\beta_0}{\beta_1}e^{\frac{\beta_1}{3}t}+C \tag{28}$$

$$v=e^{-\frac{\beta_1}{3}t}\left(\frac{\beta_0}{\beta_1}e^{\frac{\beta_1}{3}t}+C\right) \tag{29}$$

$$v=\frac{\beta_0}{\beta_1}+Ce^{-\frac{\beta_1}{3}t} \tag{30}$$

But $v=W^{1/3}$, hence, equation (30) becomes

$$W^{1/3}=\frac{\beta_0}{\beta_1}+Ce^{-\frac{\beta_1}{3}t} \tag{31}$$

$$W=\left(\frac{\beta_0}{\beta_1}+Ce^{-\frac{\beta_1}{3}t}\right)^3 \tag{32}$$

Thus, the total weight of the fish at any time t is given by the following equation

$$W(t)=\left(\frac{\beta_0}{\beta_1}+Ce^{-\frac{\beta_1}{3}t}\right)^3 \quad (33)$$

But $\beta_0=\kappa M$ and $\beta_1=\kappa N$, thus we have the following equation as the solution to the Bernoulli's differential equation in equation (12).

$$W(t)=\left(\frac{M}{N}+Ce^{-\frac{\kappa N}{3}t}\right)^3 \quad (34)$$

where $C=W_0^{\frac{1}{3}}-\frac{M}{N}$

5. RESULTS

The mathematical model defined in equation (12) above was solved analytically using Bernoulli's method of solving ordinary differential equations. The model was simulated using Wolfram Mathematica, to demonstrate the influence of the length-weight relationship on the growth of a fish species. For the numerical computations, the values of the parameters were given by the following $\kappa_1=1, \kappa_2=2, \kappa_3=3, \kappa_4=4, M=3, N=2, w_0=0, t_0=0$ and $t_f=5$ for the parameters that constitute the constants of proportionality in the system (the model). These values constitute the plot in Fig.1 below and the numerical results are found in Table1 below. However the parameter t_f equals twenty five for the results in Table 1 equals fifty for those (results) in Table 2 and equals twenty for those in Table 3. The values of the parameters that constitute the plot in Fig.2 and the results in Table2 below are $N_1=1, N_2=2, N_3=3, N_4=4, M=3, \kappa=1, w_0=0, t_0=0$ and $t_f=5$. Finally, Fig.3 and Table 3 are constituted by the following parameters $M_1=1, M_2=2, M_3=3, M_4=4, N=3, \kappa=1, w_0=0, t_0=0$ and $t_f=5$.

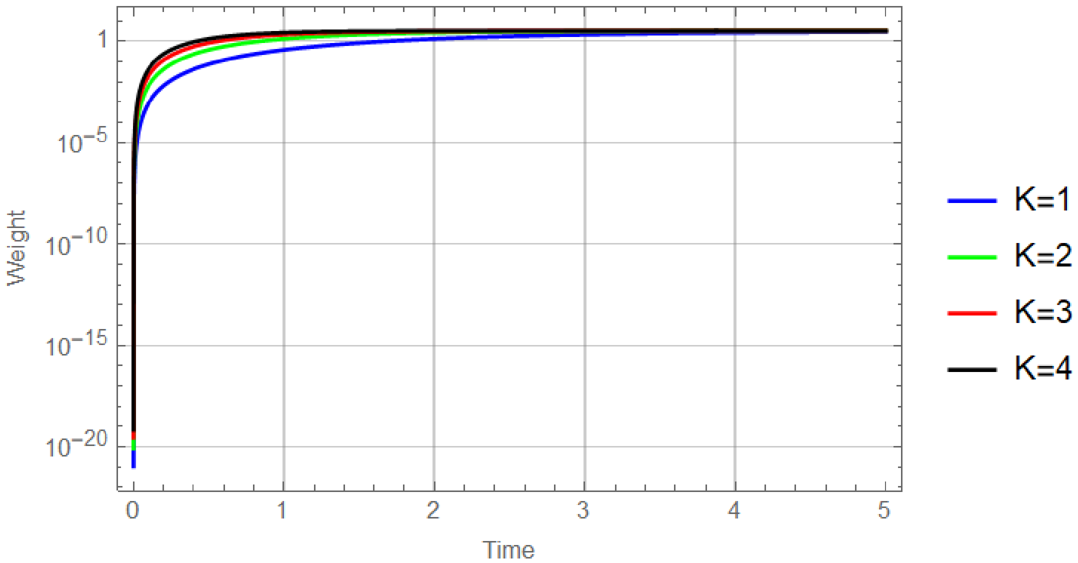

Fig.1 Variation in weight of a fish species over time with $\kappa_1=1, \kappa_2=2, \kappa_3=3, \kappa_4=4, M=3, N=2$

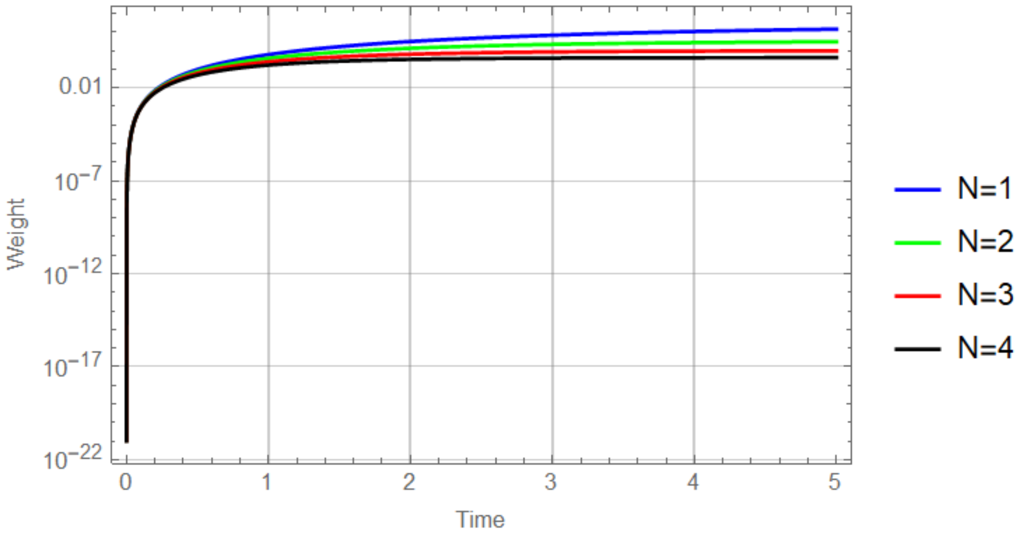

Fig. 2 Variation in weight of a fish species over time with $N_1 = 1,\ N_2 = 2,\ N_3 = 3,\ N_4 = 4,\ M = 3,\ \kappa = 1$

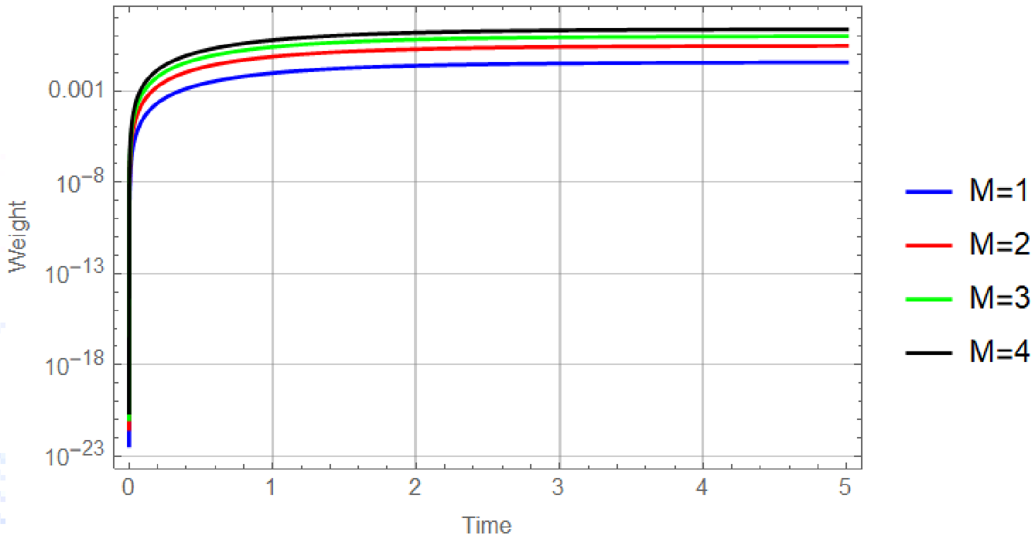

Fig. 3 Variation in weight of a fish species over time with $M_1 = 1,\ M_2 = 2,\ M_3 = 3,\ M_4 = 4,\ N = 3,\ \kappa = 1$

Table 1 Variation in weight of a fish species over time $\kappa_1 = 1,\ \kappa_2 = 2,\ \kappa_3 = 3,$ $\kappa_4 = 4,\ M = 3,\ N = 2$

Time	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 4$
0.	0.	0.	0.	0.
1.	0.388816	1.34778	2.18181	2.71923
2.	1.34778	2.71923	3.19293	3.32635
3.	2.18181	3.19293	3.34996	3.3716
4.	2.71923	3.32635	3.3716	3.37476
5.	3.02653	3.36213	3.37454	3.37498
6.	3.19293	3.3716	3.37494	3.375
7.	3.28068	3.3741	3.37499	3.375
8.	3.32635	3.37476	3.375	3.375
9.	3.34996	3.37494	3.375	3.375
10.	3.36213	3.37498	3.375	3.375

11.	3.36839	3.375	3.375	3.375
12.	3.3716	3.375	3.375	3.375
13.	3.37326	3.375	3.375	3.375
14.	3.3741	3.375	3.375	3.375
15.	3.37454	3.375	3.375	3.375
16.	3.37476	3.375	3.375	3.375
17.	3.37488	3.375	3.375	3.375
18.	3.37494	3.375	3.375	3.375
19.	3.37497	3.375	3.375	3.375
20.	3.37498	3.375	3.375	3.375
21.	3.37499	3.375	3.375	3.375
22.	3.375	3.375	3.375	3.375
23.	3.375	3.375	3.375	3.375
24.	3.375	3.375	3.375	3.375
25.	3.375	3.375	3.375	3.375

Table 2 Variation in weight of a fish species over time $N_1 = 1,\ N_2 = 2,\ N_3 = 3,$ $N_4 = 4,\ M = 3,\ \kappa = 1$

Time	$N=1$	$N=2$	$N=3$	$N=4$
0.	0.	0.	0.	0.
1.	0.615006	0.388816	0.25258	0.168473
2.	3.11053	1.34778	0.646462	0.339904
3.	6.81967	2.18181	0.857952	0.399116
4.	10.7823	2.71923	0.946053	0.415794
5.	14.4087	3.02653	0.979922	0.420266
6.	17.4545	3.19293	0.992582	0.421451
7.	19.8823	3.28068	0.997267	0.421763
8.	21.7538	3.32635	0.998994	0.421846
9.	23.1647	3.34996	0.99963	0.421867
10.	24.2123	3.36213	0.999864	0.421873
11.	24.982	3.36839	0.99995	0.421874
12.	25.5434	3.3716	0.999982	0.421875
13.	25.9509	3.37326	0.999993	0.421875
14.	26.2455	3.3741	0.999998	0.421875
15.	26.4579	3.37454	0.999999	0.421875
16.	26.6108	3.37476	1.	0.421875
17.	26.7208	3.37488	1.	0.421875
18.	26.7997	3.37494	1.	0.421875
19.	26.8564	3.37497	1.	0.421875
20.	26.897	3.37498	1.	0.421875
21.	26.9262	3.37499	1.	0.421875
22.	26.9471	3.375	1.	0.421875
23.	26.9621	3.375	1.	0.421875
24.	26.9728	3.375	1.	0.421875
25.	26.9805	3.375	1.	0.421875
26.	26.9861	3.375	1.	0.421875
27.	26.99	3.375	1.	0.421875

28.	26.9928	3.375	1.	0.421875
29.	26.9949	3.375	1.	0.421875
30.	26.9963	3.375	1.	0.421875
31.	26.9974	3.375	1.	0.421875
32.	26.9981	3.375	1.	0.421875
33.	26.9986	3.375	1.	0.421875
34.	26.999	3.375	1.	0.421875
35.	26.9993	3.375	1.	0.421875
36.	26.9995	3.375	1.	0.421875
37.	26.9996	3.375	1.	0.421875
38.	26.9997	3.375	1.	0.421875
39.	26.9998	3.375	1.	0.421875
40.	26.9999	3.375	1.	0.421875
41.	26.9999	3.375	1.	0.421875
42.	26.9999	3.375	1.	0.421875
43.	27.	3.375	1.	0.421875
44.	27.	3.375	1.	0.421875
45.	27.	3.375	1.	0.421875
46.	27.	3.375	1.	0.421875
47.	27.	3.375	1.	0.421875
48.	27.	3.375	1.	0.421875
49.	27.	3.375	1.	0.421875
50.	27.	3.375	1.	0.421875

Table 3 Variation in weight of a fish species over time wit $M_1 = 1, M_2 = 2, M_3 = 3, M_4 = 4, N = 3, \kappa = 1$

Time	$M = 1$	$M = 2$	$M = 3$	$M = 4$
0.	0.	0.	0.	0.
1.	0.00935483	0.0748387	0.25258	0.598709
2.	0.023943	0.191544	0.646462	1.53236
3.	0.031776	0.254208	0.857952	2.03366
4.	0.035039	0.280312	0.946053	2.2425
5.	0.0362934	0.290347	0.979922	2.32278
6.	0.0367623	0.294098	0.992582	2.35279
7.	0.0369358	0.295486	0.997267	2.36389
8.	0.0369998	0.295998	0.998994	2.36799
9.	0.0370233	0.296187	0.99963	2.36949
10.	0.037032	0.296256	0.999864	2.37005
11.	0.0370352	0.296281	0.99995	2.37025
12.	0.0370364	0.296291	0.999982	2.37033
13.	0.0370368	0.296294	0.999993	2.37035
14.	0.0370369	0.296296	0.999998	2.37036
15.	0.037037	0.296296	0.999999	2.37037
16.	0.037037	0.296296	1.	2.37037
17.	0.037037	0.296296	1.	2.37037
18.	0.037037	0.296296	1.	2.37037
19.	0.037037	0.296296	1.	2.37037
20.	0.037037	0.296296	1.	2.37037

6. DISCUSSION

The results in Fig 1 and Table 1 reveal that as the constant of proportionality κ between the weight and the difference between the two energies (i.e., the energy used by the fish in seeking food (E_0) and the energy provided by the food (E_1) increases, so does the rate at which the weight of the fish species approaches its peak. It has also been discovered that the maximum weight that the fish species can acquire is approximately 3.375 units. These findings indicate that the constant of proportionality κ is a scaling coefficient for the weight of the fish species, and its value is typically determined using available length-weight data or by an experimental process.

In equation (5), N is the constant of proportionality between the energy expended by the fish in seeking food and its (the fish's) weight, whereas in equation (12), M is the constant of proportionality for the energy supplied by the food and the surface area of the fish, which is two-thirds power, the weight of the fish. Meanwhile, the results in Table 2 shows that increasing the value of N (i.e., the proportionality constant for the energy used by the fish in seeking food and its weight) limits the fish species' growth rate. As shown in Fig 2, the rate of convergence into the peak at which the growth can achieve is practically low at N=4 when compared to N=1, 2, or 3. Thus, limiting the energy expended in seeking food while increasing the energy provided by food allows the fish to grow steadily to a higher value before reaching its maximum weight, as shown in Table 2, where the maximum weight is 27units at N=1, 3.375 units at N=2, 1 unit at N=3, and 0.421875 units at N=4.

The results in Fig 3 shows that as the constant of proportionality M (the scaling factor for the energy provided by the food) increases so does the rate at which the fish's weight grows to reach its maximum weight; with higher values of M indicating a faster rate and higher measures of growth attained. These findings reveal a remarkable conformity with physical phenomena, particularly regarding that for a living organism to grow, it requires a constant supply of energy provided by food intake, as well as ensuring that the energy used to do work does not exceed the necessary energy supplied by food. If the amount or quality of feed available is limited, the fish may not grow, lose weight, or even die as a result of lack of food. Only after the maintenance requirements are met will growth occur. These requirements increase as water temperature and fish activity increase. They are more common in smaller fish than larger fish.

7. CONCLUSION

Based on the results of the model solutions and simulations, we conclude that: (1) limiting the quantity and quality of feed will limit the energy required for maintenance and expenditure on necessary and desirable physical activities, resulting in weight loss or even death of the fish species; energy requirement is the amount of food energy required to balance energy expenditure in order to maintain body size, body composition, and a level of necessary and desirable physical activity consistent with long-term good health. This includes the energy required for the fish species' optimal growth and development, United Nations University, & World Health Organization (2004). (2) As seen in the results above, fish weight growth is strongly dependent on food intake, and limiting the energy supplied by food causes retarded growth in the fish species. (3) Once a fish species' growth has reached its peak, even a steady supply of energy by feed would have almost no effect on its growth. (4) The length-weight relationship is a link between mathematics and the evaluation of variation in two variables, length and weight. When the length of the fish is known, the length-weight relationship can be used to calculate the biomass of the fish sample.

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DEFINITION OF VARIABLES AND PARAMETERS

κ Constant of proportionality that serves as a scaling coefficient for the weight of the fish species.

W Weight of the fish species under consideration.

M Constant of proportionality for the energy supplied by the food and the surface area of the fish.

N Constant of proportionality for the energy used by the fish in seeking food and its weight.

A Surface area of the fish species under consideration.

l Length of the fish species under consideration.

k Constant of proportionality that serves as a scaling factor for the weight at the length of the fish.

E_0 Energy used by the fish in seeking food.

E_1 Energy provided by food supplied.