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A Direct Method for Solving The Volterra Integral Equation Of The First Kind Using The Black Bales Function Matrix BPF

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Abstract: This research deals with a method directly And effective to solve equation Volterra Complementarity from Type the first$VK1$.And that using Minions Black BalzBlock-Pulse Functions Which is called for shortBPFAnd matrix Processes Complementarity Private With it, The main idea in this method she shorthand The equation Complementarity from Type the first And convert it to sentence Trigonometric Bottom Sin maybe solve it directly on road replacement (compensation) Front.will Complete presentation some Examples To clarify efficiency And accuracy method proposed.

Keywords: *Equation Volterra Complementarity From Type The First, Sentence Trigonometry, Minions Black Palsblock-Pulse, Formulas Actinomycetes, Matrix Processes.*

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Introduction

Has always been For Volterra equationsComplementarity plays an active role in applied mathematics and many areas of physics, where methods based on...Volterra equationsComplementarityOf the first type$VK1$In solving many practical issuesand physical applications, for example, heat conduction issues[2]and diffusion issues[3],Integral equations are often reduced to integral equations of the first kind,This research deals withA new, effective and direct way to solveVolterra integral equations of the first kind. It was organizedthis searchAs follows:SectionThe first represents an introduction to integral equations of the first type, after which we move on to studying functionsBPF and its integral process matrix inSectionSecond, and inSectionthe thirdIts process matrix is utilized to proposeBPFA direct method for solving the aforementioned integral equations and we apply it to some numerical examples to prove the accuracy and effectiveness of the proposed method inSectionthe fourth,And the oathThe fifth and last represents the conclusion of our topic, recommendations and conclusionsAnd the extent of the pros and consmethodpain!Suggestion.

Materials and Methods

1. Integral equations of the first kind

An integral equation in mathematics is an equation in which an unknown, undefined function appears next to the integral sign. There is a great connection between integral equations and differential equations, and in some problems they can be reformulated.With oneThe two methods, the most common type in these equations are Volterra integral equations of the first type. They were named after the scientist Vito Volterra, which he presented to the public in 1908, and they are of the following form:

$$\int_0^t k(t,s)\, x(s) ds = f(t); t \geq 0 \qquad (1)$$

Where and$k(t,s)$ $f(t)$*They are functions of information that can be differentiated.*

This type of equation is very important because physical problems are often reduced to integral equations of the first kind.

Integral equations of the first kind are inherently poorly conditional problems, meaning that the solution is generally unstable, and very small changes in the inputs to the problem can lead to very different results.[4]. This poor condition makes numerical solutions very difficult, as a small error in the input may lead to a large change in the numerical output. To overcome this, different regularization methods have been proposed in[1]. Some methods are usedMinionsBasis and transformation of the equationComplementaritytosentencelinearH. For equationsComplementarityfrom a kind ofthe first,It usually isFor sentencesThe obtained linearization of a large number of conditions must be solved by methodYSuitable for exampleCG, PCG, etcOf the methods mentioned in [5]And[6],These methods are very difficult to apply and the number of operationsIn whichhighvery.

Operations matrix for Black Pals minionsBPF It has a pivotal roleIn this research and through it It has been reduced Volterra equation Complementarity From the first type tosentencetriangleYehlinearHLowerHIn conditionpolicewomanGood algebraic equations that can be solved directly.In the next section,We briefly describe some propertiesBPF and matrixIts integrations.

2.Minions**BPF and matrix integrations: [8]**

A set of methods is defined$m$BPFOn the domain[0,T)As follows:

$$\phi_i(t) = \begin{cases} 1, & \frac{iT}{m} \le t < \frac{(i+1)T}{m}, \\ 0, & otherwise \end{cases} \quad (2)$$

where$i = 0.1.\dots.m-1$AndmA *positive integer, and we also assume that* $h = T/mAnd\emptyset_i$sheBPFThe same rank$i$In this research, we will assume that$T = 1$And there will be consequencesBPFKnowledge of the field and .[0,1)$h = 1/m$

MinionsBPFIt has many properties, the most important of which are segmentation, orthogonality, and completeness. It can be definedBPFExplicitly infer the segmentation property:

$$\phi_i(t)\phi_j(t) = \begin{cases} \phi_i(t), & i = j \\ 0, & i \ne j \end{cases} \quad (3)$$

where $.i.j = 0.1.\dots.m-1$

The other property, orthogonality, is defined as:

$$\int_0^1 \phi_i(t)\phi_j(t)dt = h\,\delta_{ij}\,,$$

where is Kronecker's delta.δ_{ij}

Functions can be definedBPFIn the vector form, if we assume the first phase of the functions$m$BPFWe write it briefly as follows:

$\Phi(t) = [\phi_0(t), \phi_1(t), \dots, \phi_{m-1}(t)]^T, \quad t \in [0,1)$

Taking advantage of the above properties, it can be expressed as:

$$\Phi(t)\Phi^T(t) = \begin{pmatrix} \phi_0(t) & 0 & \cdots & 0 \\ 0 & \phi_1(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_{m-1}(t) \end{pmatrix} (4)$$

$\Phi^T(t)\Phi(t) = I,$

$\Phi(t)\Phi^T V = \tilde{V}\Phi(t),$

Where she is$V$mbeam and . Moreover, we can clearly conclude that for every matrix$\tilde{V} = diag(V)$BOf rank$m \times m$He is$\phi^T(t)B\phi(t) = \hat{B}^T\phi(t)$where$\hat{B}$RepresentmA vector with elements equal to the diagonal entries of the matrixB.

MinionsBPFThey are extensible methods, as the method is on the domain and it verifies$f(t)$[0,1) $\phi_i(t)$ *It may be written compactly as follows:*

$f(t) \simeq \sum_{i=0}^{m-1} f_i\phi_i(t) = F^T\Phi(t) = \Phi^T(t)F$, (5)

Where and$F = [f_0, f_1 \dots, f_{m-1}]^T f_i$

Now, suppose that$k(t,s)$A function with two variables is differentiable over the domain, then it can be expanded similarly to achieve[0,1)BPFIn the form where$k(t,s) \approx \phi^T(t)K\,\psi(s)\phi(t)$They are vectors of

functions$\psi(s)$BPFThe same dimensionm_1And in order. is a matrix of Black Bales coefficients of rank where where is represented by where$m_2 K m_1 \times m_2 K_{ij} i = 0,1 \dots, m_1 - 1$And
$j = 0,1 \dots, m_2 - 1$As follows:
$k_{ij} = m_1 m_2 \int_0^1 \int_0^1 k(t,s)\phi_i(t)\psi_j(s)dt\, ds$(6)
For ease we will put .$m = m_1 = m_2$
To explain the operations matrix of the Black Bales functions, we will proceed to the integration calculation as follows:$\int_0^t \phi_i(\tau)d\,\tau$

$$\int_0^t \phi_i(\tau)d\tau = \begin{cases} 0, & t < ih, \\ t - ih, & ih \le t < (i+1)h, \\ h, & (i+1)h \le t < 1. \end{cases} \quad (7)$$

We note here that$t - ih$equal to$h/2$ As the midpoint of so we can approximate for by .$[ih,(i+1)h] t - ih\ ih \le t < (i+1)h h/2$
Now, it can be expressed with respect to$\int_0^t \phi_i(\tau)d\,\tau$BPFIn the formula

$$\int_0^t \phi_i(\tau)d\tau \simeq \left[0, \dots, 0, \frac{h}{2}, h, \dots, h\right]\Phi(t) \quad (8)$$

Where is the component number, so it is called the matrix of integration operations, which can be expressed as follows:$h/2\ i \int_0^t \phi(\tau)d\,\tau \approx P\,\phi(t)\, P_{m\times m}$

$$P = \frac{h}{2}\begin{pmatrix} 1 & 2 & 2 & \dots & 2 \\ 0 & 1 & 2 & \cdots & 2 \\ 0 & 0 & 1 & \cdots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \quad (9)$$

Therefore, the integral of each function can be approximated$f(t)$ As follows:

$$\int_0^t f(\tau)d\tau \simeq \int_0^t F^T\Phi(\tau)d\tau \simeq F^T P\Phi(t).(10)$$

3.A direct algorithm for solving the Volterra integral equation of the first kind

The results obtained in the previous section were usedto constructAn effective and simple direct method for solving Volterra equationsComplementarityOf the first type.

Suppose we have the Volterra integral equation of the first kind as follows:

$$\int_0^t k(t,s)x(s)ds = f(t) \quad ; 0 \le t < 1(11)$$

where$f$And the functions are known, but the function is unknown. Furthermore it,$kxk(t,s)$*And functions are differentiable on the domain.*$f(t) 0 \le t, s < 1$

By approximating the functions,fk*And as appropriate*x*BPFOur produce*

$$f(t) \simeq F^T\Phi(t) = \Phi^T(t)F,$$
$$x(t) \simeq X^T\Phi(t) = \Phi^T(t)X,$$
$$k(t,s) \simeq \Phi^T(t)K\Phi(s),$$

Where the x-raysF,XAnd$K$They are transactionsBPFFor functions, respectively, and is the vector of unknowns.$f(t)x(s)k(t,s)X$

Now substitute the previous approximations into the Volterra integral equationFrom the first type we have:

$F^T\Phi(t) \simeq \int_0^1 \Phi^T(t)K\Phi(s)\Phi^T(s)X\,ds$ (12)

$$\simeq \Phi^T(t)K\int_0^1 \Phi(s)\Phi^{\mathrm{T}}(s)X\,ds.$$

By taking advantage of Eq

$\Phi(t)\Phi^T(t)V = \tilde{V}\Phi(t)$, (13)

We get

$$F^T\Phi(t) \simeq \Phi^T K\int_0^1 \tilde{X}\Phi(s)ds$$

$\simeq \Phi^T(t)K\,\tilde{X}\int_0^1 \Phi(s)\,ds$.(14)

Taking advantage of the process matrix$P$described previously, we have:

$F^T\Phi(t) \simeq \Phi^T(t)K\tilde{X}P\Phi(t)$(15)

where is a rank matrix$K\tilde{X}Pm \times m$.

The equation$\phi^T(t)B\phi(t) = \hat{B}^T\phi(t)$*Lead us to*

$$\Phi^T(t)K\tilde{X}P\,\Phi(t) = \hat{X}^T\,\Phi(t)(16)$$

where is a vector with elements equal to the diagonal entries of the matrix$\hat{X}mK\tilde{X}P$.

So we can write the vector as follows:$\hat{X}$

$$\hat{X} = \begin{pmatrix} \frac{h}{2}k_{0,0}x_0 \\ hk_{1,0}x_0 + \frac{h}{2}k_{1,1}x_1 \\ hk_{2,0}x_0 + hk_{2,1}x_1 + \frac{h}{2}k_{2,2}x_2 \\ \vdots \\ hk_{m-1,0}x_0 + hk_{m-1,1}x_1 + \cdots + \frac{h}{2}k_{m-1,m-1}x_{m-1} \end{pmatrix}(17)$$

We can also write it in the form:

$$\hat{X} = h\begin{pmatrix} \frac{1}{2}k_{0,0} & 0 & 0 & \dots & 0 \\ k_{1,0} & \frac{1}{2}k_{1,1} & 0 & \dots & 0 \\ k_{2,0} & k_{2,1} & \frac{1}{2}k_{2,2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_{m-1,0} & k_{m-1,1} & k_{m-1,2} & \dots & k_{m-1,m-1} \end{pmatrix}\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{m-1} \end{pmatrix}(18)$$

From the above we arrive at the result, or in another mathematical form we can write it as follows:$\hat{X} - F = 0$

$h\sum_{j=0}^{i\,\prime} k_{ij}\,x_j = f_i\quad ; i = 0.1 \dots m-1$ (19)

Where it means that the last entry owns a worker$\sum' \frac{1}{2}$.

If the Volterra integral equation of the first kind has a single solution, then the set of equations we obtained has a good condition and represents a set of minimum trigonometric linear equations consisting of an algebraic equation and an unknown. mm
Which can be solved very easily by the substitution method (forward substitution), then the solution can be approximated and calculated to the Volterra integral equation of the first type that is required to be solved, and all of this is done without using any method of projection, and this is one of the advantages of our proposed method, as it is simple, cheap, and reduces calculations. Significantly. $x_i \quad ; i = 0.1 \ldots . m - 1 x(t) \approx X^T \phi(t)$

4.The applied aspect

This proposed method will be applied to numerical examples selected from various sources in order to enable usCompare the numerical results obtained here with both the exact solution and the resultsNumericalThe other. The calculations associated with the examples were performed usingMATLAB.

Example1 Suppose we have the following integral equation.[7]

$$\int_0^t \cos(t-s)\, x(s) ds = t \sin\ t$$

Where the exact solution is and so for$x(t) = 2 \sin t \; 0 \le t < 1$

The numerical results of the exact and approximate solution for different values of and are shown in Table 1. tm

Table 1: Results of the proposed method, exact and approximate solution of Example 1 for different values of and $.tm$

t	*Exact solution*	*Approximate solution* $m = 64$	*Approximate solution* $m = 128$
0	0	*0.010417*	*0.005208*
0.2	*0.397339*	*0.382942*	*0.389412*
0.5	*0.958851*	*0.967335*	*0.963098*
0.7	*1.288435*	*1.276056*	*1.289847*
0.9	*1.566654*	*1.569934*	*1.572171*

These results have good accuracy compared to the numerical results obtained usinginstallation betweenroadAssembly slices Spline-Collocation and Lagrange interpolation which we see in [6].

Example2 Let us have the following Volterra integral equation of the first kind:

$$\int_0^t e^{t+s}\, x(s)ds = t\, e^t$$

The numerical solutions resulting from solving this equation using the proposed method are presented in Table 2 for different values of and .tm

Table 2:*results method proposed, the solution Flour And the approximate For example2 from Okay Valuable Different For tAnd m.*

t	Exact solution	Approximate solution $\mathbf{m = 32}$	Approximate solution $\mathbf{m = 64}$
0	*1*	*0.989584*	*0.994792*
0.1	*0.904837*	*0.891689*	*0.905768*
0.3	*0.740818*	*0.739236*	*0.735426*
0.5	*0.606531*	*0.600213*	*0.603372*
0.7	*0.496585*	*0.497594*	*0.500213*
0.9	*0.406570*	*0.412520*	*0.406141*

We note from the numerical results that our proposed method has greater effectiveness in solving the Volterra integral equation of the first kind compared to many other methods, in addition to what was previously mentioned about the extent to which it reduces the number of mathematical operations that are performed, and this has a positive effect on accuracy and efficiency.

Results and Discussion

In this paper, a straightforward and effective method based on BlackBals functions is presentedBPFTo solve the Volterra integral equations of the first type, which contribute to solving many problems in mathematical physics, applied engineering, and other fields. This method aims to transform the Volterra integral equation of the first type into a minimum linear trigonometric system of algebraic equations that are easily solved by substitution methods. And substitution, we have provided some examples through which we were able to evaluate the proposed algorithm and the extent of its accuracy and efficiency in solving this type of equations. The numerical results have proven the superiority of this method based on functions.VK1BPFCompared to other methods in finding the closest numerical solution to the exact solution and also reducing the number of mathematical operations used.

Conclusion

1. It should be noted that in the middle of each subdomain the approximate solution will be more accurate, and this accuracy will increase as .$[ih\,,(i\ +\ 1)\ h]m$
2. Some points far from the middle of the field may oscillate as the value increases, but these oscillations are of course negligible, and this can be clearly followed through the matrix of integrals of the functions.mPBPF.

3. The benefits of this method are the low cost of preparing the equations without applying any projection method such asGalerkin,Assembly, Lagrange interpolation And others. also!,SentencelinearHHYTriangle systemY LowerIt can be easily solved by anterior replacementAndLow number of operations cDra!.

finallyHere are the recommendations In itThis method can be easily extended and applied to systems of integral Volterra equations of the first kind.

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